M1. Turning triangles – solutions and what to look for

Solutions

Part 1

(a) a = 72°, b = 54°

Notes: This question is designed to be straightforward. It gives children the confidence to move on.

(b) 36°, 72°, 72°

(c)

Number of sides	3	4	5	6	9	10	12	15	18	20	30	36
Angle a	120	90	72	60	40	36	30	24	20	18	12	10
Angle b	30	45	54	60	70	72	75	78	80	81	84	86

Part 2

(a) c = 72°, d = 36°

Notes: Children may be encouraged to consider why two of the angles in this triangle are the same as the triangle in part 1(b).

(b) c = 72°, d = 36°
h = 108°, j = 54°,
k = 18°

Notes: Children may find it useful to refer back to part 1(a).

(c) $d = 180 - 2c$

$h = 180 - c$

$j = \frac{1}{2}(180 - c)$

$k = j - d$

Notes: Other ways of expressing the relationships are possible. Some children will wish to investigate specific polygons before considering how to be certain that their findings will always work. Other children will be able to use algebra to prove that the relationships must always hold.

Reviewing mathematical achievement

Level 5

Typically children working at level 5 are able to apply their knowledge of angle properties to identify angles within the triangles. In part 1(c), they work within the given constraints and check their results, considering whether their solutions are sensible. They understand that identifying 12 polygons is insufficient to constitute certainty that no other such polygons exist. Their mathematical communication, both written and oral, is clear enough that others can follow their logic.

In part 2 of the task, they can apply spatial reasoning to understand how the windmills are constructed, and in part (a) can work out correctly the values of angles c and d. They express simple relationships between the angles, for example $h + c = 180$.

Level 6

Typically children working at level 6 are able, in part 1(c), to give a reasoned argument as to how they know there are no more than 12 polygons. They show evidence of being systematic and logical, for example by considering factors in order with explanations as to why factors are accepted or rejected.

In part 2 of the task, they apply geometrical reasoning to work out the values of all the angles in the regular pentagon. They show understanding of the given diagram by reasoning generally for other windmill patterns, and they are able to establish more complex relationships between the angles, for example $h = 2j$, even if they need encouragement to express these relationships using formal algebra.

M2. Award **ONE** mark for the correct answer of 108°

Award **ONE** mark for appropriate explanation, eg:

- 180 –72
- regular pentagon, angles are 108°
- isosceles triangles, 2 × 54

Up to 2

[2]

M3. (a) 40°

1

(b) 25°

1

[2]

M4. Award **TWO** marks for the correct answers x = 125 **AND** y = 145.

If the answers are incorrect award **ONE** mark for either x = 125 **OR** y = 145 **OR** the sum of x and y being 270.

up to 2

[2]

M5. 132

[1]

M6. (a) $\frac{1}{9}$ **OR** 0.1 recurring **OR** $11\frac{1}{9}$ %

Accept equivalent fractions eg $\frac{40}{360}$

Accept 0.11 or 11%.

***Do not** accept answers in words, eg '1 out of 9' **OR** '1 in 9'*
***OR** ratios, eg 1:9*

1

(b) Explanation which recognises that all the numbers are not equally likely to come up because the angles formed at the centre by each section are not equal, eg

- 'Some are narrower at the centre than others';
- 'The angles in the centre aren't equal'.

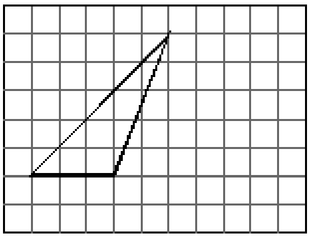
***Do not** accept vague or arbitrary explanations such as*

- *'It's just luck';*
- *'Some have more space';*
- *'They never are equal'.*

1

[2]

M7. Award **TWO** marks for any obtuse-angled triangle with an area of 7.5cm^2, eg



If the answer is incorrect, award **ONE** mark for any triangle with an area of 7.5cm^2

(irrespective of angles)

Accept any obtuse-angled triangle with appropriate base and height each correct to within 2mm

The triangle need not have vertices on the grid intersections.

Accept a triangle not drawn with a ruler, provided the vertices are correctly placed.

Up to 2

[2]

M8. (a) 98

1

(b) T = 2R – 2

OR

R = $\frac{T+2}{2}$

Accept equivalent expressions, eg
T = R × 2 – 2
T = 2 × (R –1)
R = $\frac{T}{2}$ *+ 1*

Accept answers in words, eg

- *'to get T, you times R by 2 and then you take away 2';*
- *'it's 1 less than R, then you double it and that's T'.*

1

[2]

M9. (a) 55°

If answers for 9a and 9b are transposed, but otherwise correct, award the mark for 9b only

1

(b) 25°

1

[2]

M10. Award **TWO** marks for the correct answer of 150°

If the answer is incorrect, award **ONE** mark for evidence of an appropriate method, eg

360 ÷ 36 = 10
15 × 10

Calculation need not be completed for the award of the mark.

Up to 2

[2]

M11. (a) *x* = 155°

1

(b) *y* = 85°

*If answers for 5a and 5b are transposed, but otherwise correct, award **ONE** mark only, in the 5b box.*

1

[2]

M12. (a) *x* = 55°

1

(b) *y* = 20°

OR *y* = (Answer to (a) – 35°)

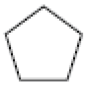
*If answers to x and y are transposed but otherwise correct, award **ONE** mark only in the (b) box.*

1

[2]

M13. Indicates No and gives a correct explanation

eg

- The angles are not the same size
- A regular pentagon looks like this,  with its angles all the same size
- All the angles should be 108°
- It doesn't have rotation symmetry
- It's got more sides than a square so all its angles should be obtuse, but they're not

1

60°

2

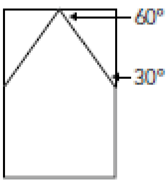
Shows that the 150° angle can be split into 90° and 60°

or

Divides the pentagon vertically and shows that half a is 30°

or

Draws triangles to show a rectangle, labelling the non-right angles on at least one side correctly eg

- 

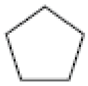
or

Shows or implies that the angle sum of a pentagon is 540° **1**

Accept minimally acceptable explanation eg • 90 ≠ 150
• Different angles • A regular pentagon doesn't have right angles in it • A regular one can't have 150° angles • It doesn't look the same when it's turned
• Not all the angles are obtuse

! Incorrect angle size for a regular pentagon given Condone alongside a correct response eg, accept • The angles are different, they should be 60° (error, but all equal implied)
• The angles should all be 70° (error) eg, do not accept
• The 90° angles should be 60° (does not imply the angles should all be the same)

***Do not accept** incomplete explanation eg • Not the same • It has two right angles • Two angles are the same*

• A regular pentagon looks like this 
• A regular pentagon doesn't have any vertical lines

! Indicates Yes, or no decision made, but explanation clearly correct
Condone provided the explanation is more than minimal

[3]

M14. b = 50 **1**

a = 20 **1**

U1

As evidence of a correct method, in either part, shows or implies that the angles in one of the triangles are a, b and b

eg, in the first question part

- 80, 50, 50 seen
- (180 – 80) ÷ 2
- (360 – 160) ÷ 2 ÷ 2

eg, in the second question part

- 180 – 2 **×** 80
- (360 – 160 × 2) ÷ 2

eg, correct answers transposed

! Incomplete or no working shown

Provided at least one correct angle is credited, award this mark

! In the second question part 80, 80, 20 is insufficient without any indication of the position of the equal angles

1

[3]

E1. **What to look for**

Part 1

Angle properties Ma3 2a

Problem solving Ma3 1b, Ma3 1c, Ma3 1d
Reasoning Ma3, 1h, Ma2 1j, Ma2 1k

Part 1(a)

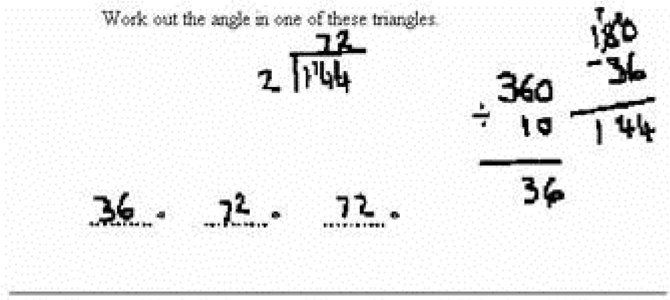
Commentary

Most children use their knowledge of angles at a point and angles in a triangle to calculate the answer. While most use paper and pencil, some use a calculator

Part 1(b)

Some children start with 360° at the centre of the polygon

Examples: 360 ÷ 10 = 36
180 – 36 = 144
144 ÷ 2 = 72

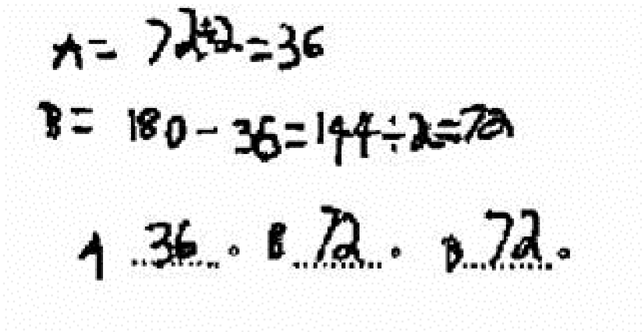


Some children use the answer to previous question to help them.

Examples: 72 ÷ 2 = 36

180 – 36 = 144

144 ÷ 2 = 72



Very able children may show evidence not only of trying to develop logical thinking, but also of using checking procedures to confirm or refute the reasoning.

Examples: In this example, the child checks an assumption and finds it to be incorrect

decagon = 10 sides, pentagon = 5 sides. Therefore triangles must be half the size so I think you can double a and half b. But we found this doesn't work.

Part 1(c)

More able children recognise that the number of triangles and the number of angles at the centre of a polygon are the same as the number of sides.

Examples: *If a shape has so many sides the amount of triangles inside the shape will be the same. Every shape has the same amount of triangles as it does have sides*

Able children quickly realise that factors of 360 are required.

When considering the constraints most children notice that a shape with more than 36 sides will result in central angles that are less than 10?. For example, 40 is a factor of 360 but a regular polygon with 40 sides gives a central angle of only 9?.

Examples: *Only 12 numbers 36 and under are factors of 360. They must be under 36 because if they were over then the inside angles would be under 10. They have to be factors of 360 for the angles to be whole numbers*

Able children also realise that factors, once found, need to be reconsidered in the context of the problem. Initially, some children may include an octagon in their list of polygons.

Although 8 is a factor of 360 (8 × 45), the other two angles of the triangles in a regular octagon are not whole numbers.

As part of the process of checking the factors of 360, some able children may realise that the angle at the centre needs to be an even number to ensure that the other two angles will be whole numbers

Examples: *All the numbers when* ??*into 360 that give an even number are one of the 12, not including 1 and 2 because you can't have a 1 or 2 sided polygon*

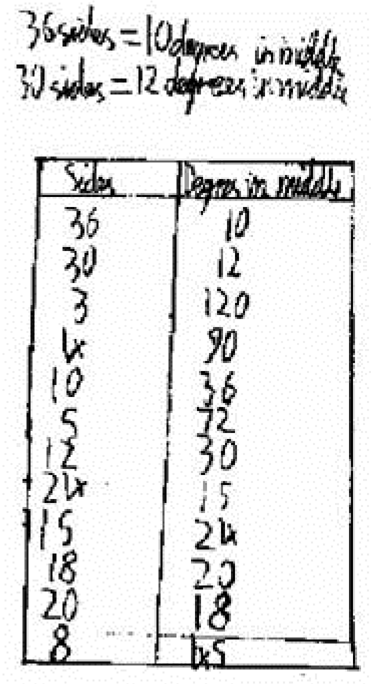
The most common error is to list factors and stop when 12 are found, even if some of those 12 are incorrect, without realising that this does not constitute proof.

Communicating Ma3 1e, Ma3 1g

Able mathematicians engage well with this part of the task. The challenge here is not only to convince themselves that there are only 12 such regular polygons, but to produce a reasonable justification for others.

In exploring the factors of 360, some children work through all the possible factors of 360, tabulating or listing their results

Example



36 sides = 10 degrees in middle
30 sides = 12 degrees in middle

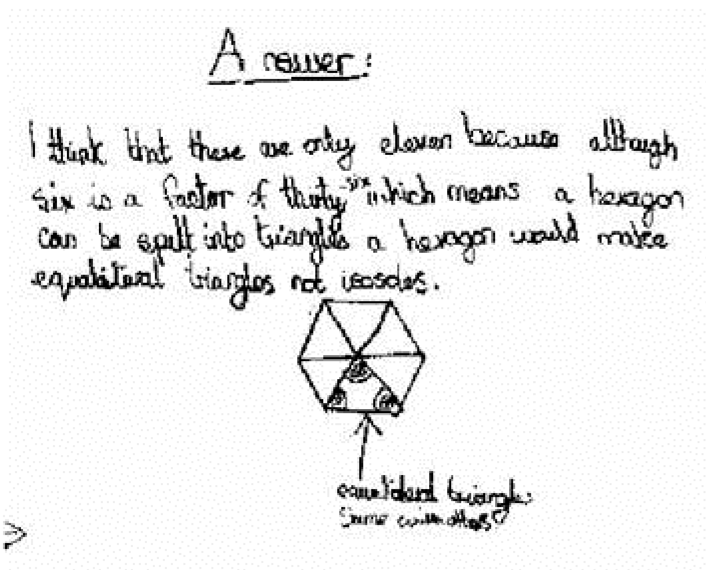
Sides	Degrees in middle
36	10
30	12
3	120
4	90
10	36
5	72
12	30
24	15
15	24
18	20
20	18
8	45

Whilst working in an exploratory way, many children start by producing work, which appears chaotic on the page.

Examples: Teacher questioning or prompting encourages children to be more systematic and to present their results in a more organised way.

Some children discount the hexagon as it is made up of equilateral triangles. Children may argue that an equilateral triangle is not an isosceles triangle (although it is a special case, as a square is a special rectangle).

Examples: The quality of the arguments in the example below is good and, despite this error, shows an awareness of the need to convince others of decisions taken.

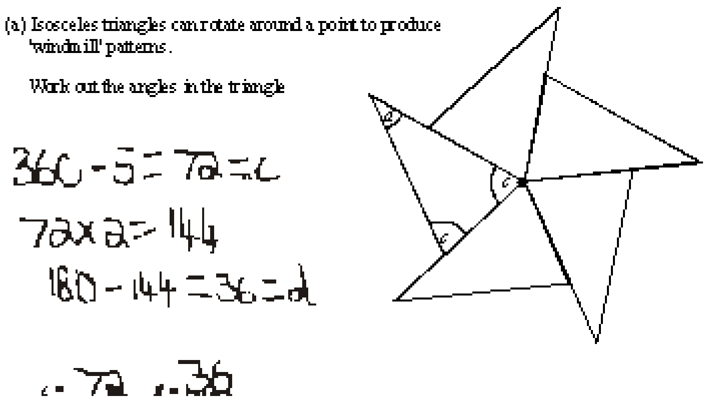


Part 2(a)

Angle properties (Ma3, 2a) and problem solving (Ma3, 1b; Ma3, 1c; Ma3, 1d)

Able children make use of their knowledge of the angles in triangles and at a point to calculate their answers.

Example



This child calculates the angles in a similar way to the method used for part 1(a).

Typically calculations are done sequentially

Examples: Not all children will do sequential calculations in this order:

- *360° in a circle. 360 ÷ 5 = 72° = c*
- *72° + 72° = 144°. 180 – 144 = 36° = d*

- *18° on a straight line. 180 – 72 = 108° = h*
- *180°72 = 108°, 108 ÷ 2 = 54° = j*
- *54 – 36° = 18° = k*

Some children realise that they are recalculating angles that have been worked out earlier and they may refer back to parts 1(a) and 2 (a). This is an efficient way of working.

Examples: *c = 72°, d* = 36° same *as before*

Part 2(c)

Most children note with confidence the relationship between angles h and c

Examples: *$h + c$ = 180 as they form a straight line*

Angles $c + h$ makes a straight line. So if you know c you would take it away from 180° to find h

Predicting angles, j and k, given c and d, is a difficult task for all but the most mathematically able

Examples: *If you know j and d you could take d away from j to find the answer to k.*

To work out k you add $h + j$ and take it away from 180°'

$c + h$ = 180°, 180 – c ÷ 2 = j, $j + h + k$ = 180°

Very able children show detailed reasoning

Examples: *In the big triangle, $k + d + c + j$ = 180°. But this big triangle is isosceles so $k + d$ must be the same as j. What I did was to put in j instead of $k + d$ so I got $j + c + j$ = 180°*

Some children do not believe it possible to predict angles *h, j & k* given *c & d*. Some may feel that they will need to draw and measure each 'windmill' to be sure

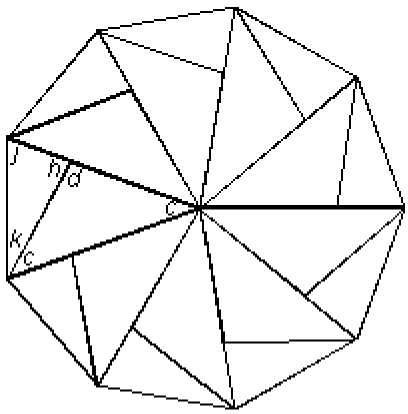
Some children, present incomplete reasoning

Examples: *It will work because the triangle would always add up to 180?*

Note:

When angle *d* is greater than 90??(as in a 9-sided polygon) the 'outside' isosceles triangle (the one shaded on the children worksheet) becomes inverted so that it is angles *d* and *h* that form 180° rather than *c* and *h*.

It is unlikely that any children will identify this as an issue. Generalisations drawn from the pentagon are acceptable within the context of this task.

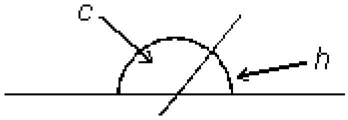


Communicating (Ma3, 1e; Ma3, 1g)

Commentary

Most children use a combination of words and simple algebraic statements, with varying amounts of justification, to communicate their ideas.

Examples: *If there are two angles*



then you take the angle you know (c)from 180° and that will give you angle h

A straight line is 180° and as c is 72°, h = 180 – 72 = 108°

Children are often able to articulate their thoughts algebraically in discussion when they are prompted by effective questioning

From this, children are sometimes able to record their understanding using simple algebra

Examples: T: *What can you tell me about this big triangle? (the triangle with angles* j*,*
$k + d$ *and* c*)*

Ch: *The angles add up to 180°.*

T: *Good, what else?*

Ch: *It is an isosceles triangle.*

T: *Okay, so what can you say now?*

Ch: *That if you add angles* d *and* k *it'll be the*

E3. No comment available.

N1. TURNING TRIANGLES

In this two-part task children explore the angles of polygons, triangles and other shapes.

Part 1: children consider regular polygons as being made of isosceles triangles that fit together around a point. They work out a set of angles within a pentagon and decagon before investigating other polygons.

Part 2: children investigate windmill patterns produced by fitting isosceles triangles together around a point. They are then encouraged to generalise rules for angles in windmill patterns enclosed by polygons

This task is intended for more able children but can be adapted to be accessible to all. To adapt the task to an appropriate level for whole class work, for example, children could be asked to find only six of the twelve polygons made from isosceles triangles that fit together around a point. Making the task more concrete by using shapes made from plastic or card may also be helpful.

PRIOR KNOWLEDGE

Children working on this task need to know or understand:

- the angles of a triangle add up to 180°;
- the angles at a point add up to 360°;
- the angles on a straight line add up to 180°;
- isosceles triangle, pentagon, decagon and regular polygon;
- an equilateral is a special case of an isosceles triangle.

Children should spend two or three hours on the task, though this may vary. Although children may complete the task in one sitting - revisiting the task after a break of hours or days allows time for reflection and is to be advised.

RESOURCES

For each child, or group of children:

- a copy of the photocopiable two-page task *pupil sheet*
- paper and pen;
- if children prefer, a ruler to draw diagrams (optional)

For the teacher:

- solutions and what to look for

WHAT TO DO

This is not a traditional test and is designed to be worked collaboratively.

Explain to children that the task has two parts. They will be exploring shapes and angles.

Part 1

- Explain to children that in Part 1 they are going to look at ways in which polygons can be made by fitting isosceles triangles around a point.
- Ensure that children read 1 (b) correctly. Errors result if children begin working with 10.
- In Part 1 (c) children will find only eleven possible polygons if they do not consider an equilateral triangle to be an isosceles triangle. However, by drawing parallels with their knowledge that a square is a special rectangle, teachers can challenge this misunderstanding.

Part 2

- Explain to children that in Part 2 isosceles triangles are arranged differently, to produce a windmill pattern. Children use what they have learned in Part 1 and what they know about angles to work out the angles in the triangles. They then consider the polygon that can be drawn by joining the exterior points of the windmill. They work out the angles and try to generalise what [rules] they have discovered for any such shape.
- Encourage children to communicate effectively, both orally and in writing. [Engage with the children to establish their mathematical reasoning and understanding. Discussion can help children explore and correct misunderstandings.]
- Encourage them to explain and justify their mathematical working, especially when the children are confident writers.

Children should be free to decide whether they want to draw diagrams to help them with this task. Many will want to do so, particularly in the early stage of trying to visualize a problem. However, drawing is not compulsory. Some children may need reassurance that drawings are not necessary. Encourage able children to calculate angles without drawing.

Part 2 (b) is more challenging than the earlier work on the task. Children many need further guidance and discussion.

Part 2 (c) invites exploration of angles in a complicated context. Even very able children may want to revert to drawing or using cut out triangles to help them. Once children have understood the activity they should be encouraged to use visualisation rather than drawing.

- Some able children should be encouraged where appropriate to work algebraically.

- When a child has a complete and well presented numerical calculation for part 2 (b), you can suggest that they substitute letters for numbers in their working. This will help with Part 2 (c).

WHAT NEXT?

It would be possible to extend the task by considering questions such as:

- How many different windmill patterns, with angles that are whole numbers greater than or equal to 10?, is it possible to make?

- Can you use a computer based geometrical drawing package to construct some of these windmill patterns?

- Consider the isosceles triangles that can form regular polygons, alongside those that can form ‘windmill’ patterns. What is the relationship between them?